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DA-003-001543

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

March – 2022

Statistics : Paper - 509

(Mathematical Statistics)

(Old Course)

Faculty Code : 003

Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Q.1 carries 20 Marks.

(2) Students can carry their own scientific calculates

1 Filling the blanks and short questions : **20**

(1) If X_1, X_2 are X_3, three variables, the regression planes X_1 on $X_2, X_3; X_2$ on X_1, X_3 and X_3 on X_1, X_3 are coincident iff ________

(2) ________ is a characteristic function of Standard Normal distribution.

(3) Given a joint Bivariate Normal distribution of X, Y as $BVN\left(\mu_x, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$, the marginal distribution $f_X(x) =$ ________

(4) ________ is a moment generationg function of Normal distribution.

(5) ________ is a moment generating function of $\gamma(\alpha, p)$.

(6) ________ is a moment generating function of Chi-square distribution.

(7) For Normal distribution $\mu_{2n} =$ ________

(8) Pearson's coefficient of skewness for Chi-square distribution curve is ________

(9) If two independent variates $X_1 \sim N\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim N\left(\mu_2, \sigma_2^2\right)$ then $X_1 - X_2$ is distributed as ________

(10) If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1+X_2}$ is distributed as ________

(11) If two independent variates $X_1 \sim \Lambda\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim \Lambda\left(\mu_2, \sigma_{22}^2\right)$ then $X_1 \cdot X_2$ is distributed as ________

(12) Wheibull distribution has application in ________

(13) If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2 respectively, then the distribution of $\frac{\chi_1^2}{\chi_2^2}$ is ________

(14) The range of multiple correlation coefficient R is ________

(15) The range of partial regression coefficient is ________

(16) Define Caush's distribution.

(17) Define Log Normal with $\log_e(x-a)$ distribution.

(18) Write mean and variance of Gama distribution with parameter (α, p).

(19) Write mean and variance of Weibul distribution.

(20) Write mean and variance of Lapace(double) exponential distribution.

2 (A) Write the answer any **three** : **6**

(1) Define Convergence in Probability.

(2) If $u = \frac{x-a}{h}, a$ and h being constants then $\varnothing_u(t) = e^{(-iat/h)} \varnothing_x(t/h)$

(3) Define Weibul distribution.

(4) Usual notion of multiple correlation and multiple regression, prove that $\sum X_{1.23} x_2 = 0$

(5) Prove that $b_{12.3} = \frac{b_{12} - b_{13} b_{23}}{1 - b_{13} b_{23}}$

(6) In trivariate distribution it is found that $r_{12} = 0.86, r_{13} = 0.65$ and $r_{23} = 0.72$.
Find $(i)\, r_{12.3}\, (ii)\, R_{1.23}$

(B) Write the answer any **three** : **9**

(1) Prove that $\mu_r = (-1)^r \left[\frac{d^r}{dt^r} \varnothing_u(t)\right]_{t=0}$;
where $u = x - \mu$

(2) Obtain Probability density function for the characteristic function $\varnothing_x(t) = p\left(1 - qe^{it}\right)^{-1}$

(3) Define Exponential distribution and obtain its MGF. From MGF obtain its mean and variance.

(4) Define truncated Poisson distribution and also obtain its mean and variance.

(5) Usual notation of multiple correlation and multiple regression, prove that $b_{12} = \frac{b_{12.3} + b_{13.2} b_{32.1}}{1 - b_{13.2} b_{31.2}}$

(6) Usual notation of multiple correlation and multiple regression, prove that $\sigma_{1.23}^2 = \sigma_1^2\left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$

(C) Write the answer any **two** : **10**

(1) If x and y are two independent random variable and its probability distribution is define as follow than obtain its characteristic function

$$f(x,y) = \begin{cases} \frac{1}{3};\ f\ or(x,y)=\{(1,1),(1,-1)\} \\ \frac{1}{6};\ f\ or(x,y)=\{(-1,1),(-1,-1)\} \\ 0;\ other\ wise \end{cases}$$

(2) Drive t-distribution.

(3) Drive χ^2 distribution and show that $2\beta_2 - \beta_1 - 6 = 0$.

(4) Obtain maginal distrbution of x for Bi-variate distribution.

(5) Usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{13}}{1 - r_{23}^2}$

3 (A) Write the answer any **three** : **6**

(1) Define Beta-I and Beta-II distribution.

(2) Obtain characteristic function of Poisson distribution with parameter λ.

(3) Define truncated distribution.

(4) Usual notion of multiple correlation and multiple regression, prove that $\sum X_{1.2}X_{3.12} = 0$

(5) Prove that $\sigma_{3.12}^2 = \dfrac{\sigma_3^2\left(1 - r_{12}^2 - r_{23}^2 - r_{13}^2 + 2r_{12}r_{23}r_{13}\right)}{\left(1 - r_{12}^2\right)}$

(6) In trivariate distribution it is found that $\sigma_1 = 2, \sigma_2 = \sigma_3 = 3, r_{12} = 0.7, r_{23} = r_{31} = 0.5$
Find $(i)\, b_{13.2}$ $(ii)\, \sigma_{3.12}$

(B) Write the answer any **three** : (Each 3 marks) **9**

(1) Prove that $\mu_r' = (-i)^r \left[\dfrac{d^r}{dt^r} \varnothing_X(t)\right]_{t=0}$

(2) Obtain MGF of Normal distribution.

(3) Obtain mean and variance of Uniform Distribution.

(4) Define truncated Binomial distribution and also obtain its mean and variance.

(5) Usual notation of multiple correlation and multiple regression, prove that If $r_{12} = r_{23} = r_{31} = r$ then
$R_{1.23} = R_{2.31} = R_{3.12} = \dfrac{\sqrt{2}r}{\sqrt{1+r}}$

(6) Usual notation of multiple correlation and multiple regression, prove that $b_{12.3}b_{23.1}b_{31.2} = r_{12.3}r_{23.1}r_{31.2}$

(C) Write the answer any **two** : (Each 5 marks) **10**

(1) Obtain MGF of Gamma distribution with parameters α and p. Also show that $3\beta_1 - 2\beta_2 + 6 = 0$.

(2) Drive Normal distribution.

(3) Drive F-distribution.

(4) Obtain conditional distribution of y when x is given for Bi-variate distribution.

(5) Usual notation of multiple correlation and multiple regression, prove that $r_{12.3} = \dfrac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1 - r_{13}^2\left(1 - r_{23}^2\right)\right)}}$